

Basic Ideas
In
Fluid Mechanics

Introduction

Understanding fluid mechanics is needed for:

- Biomechanics - To understand the flow of blood and cerebral fluid.
- Meteorology and Ocean engineering - To understand the motion of air movements and ocean currents.
- Chemical engineering - To design different kinds of chemical-processing equipment.
- Aeronautical engineering - To maximize lift, minimize drag on aircraft, and to design fan-jet engines.
- Mechanical engineering - To design pumps, turbines, internal combustion engines, etc.

Dimensions, Units, and Physical Quantities

- **Fundamental Dimensions** - Nine quantities that can express all other quantities.
 1. Length
 2. Mass
 3. Time
 4. Temperature
 5. Amount of a substance
 6. Electric current
 7. Luminous intensity
 8. Plane angle
 9. Solid angle

Dimensions, Units, and Physical Quantities

Example:

$$F = ma$$

$$[F] = [m][a]$$

$$F = M \frac{L}{T^2}$$

- There are two primary systems of units:
 - English units
 - Système International units (SI)

Dimensions, Units, and Physical Quantities

Fundamental Dimensions and Their Units

<i>Quantity</i>	<i>Dimensions</i>	<i>SI units</i>		<i>English units</i>	
Length ℓ	L	meter	m	foot	ft
Mass m	M	kilogram	kg	slug	slug
Time t	T	second	s	second	s
Electric current i		ampere	A	ampere	A
Temperature T	Θ	kelvin	K	Rankine	$^{\circ}\text{R}$
Amount of substance	M	kg-mole	kmol	lb-mole	lbmol
Luminous intensity		candela	cd	candela	cd
Plane angle		radian	rad	radian	rad
Solid angle		steradian	sr	steradian	sr

- **Derived Quantities** - Combinations of fundamental quantities to form different parameters.

Dimensions, Units, and Physical Quantities

Derived Units

<i>Quantity</i>	<i>Dimensions</i>	<i>SI units</i>	<i>English units</i>
Area A	L^2	m^2	ft^2
Volume V	L^3	m^3	ft^3
		L (liter)	
Velocity V	L/T	m/s	ft/s
Acceleration a	L/T^2	m/s^2	ft/s^2
Angular velocity ω	T^{-1}	rad/s	rad/s
Force F	ML/T^2	$kg \cdot m/s^2$	slug-ft/s ²
		N (newton)	lb (pound)
Density ρ	M/L^3	kg/m^3	slug/ft ³
Specific weight γ	M/L^2T^2	N/m^3	lb/ft ³
Frequency f	T^{-1}	hertz (cycles/s)	s ⁻¹ (hertz)
Pressure p	M/LT^2	N/m^2	lb/ft ²
		Pa (pascal)	(psf)
Stress τ	M/LT^2	N/m^2	lb/ft ²
		Pa (pascal)	(psf)
Surface tension σ	M/T^2	N/m	lb/ft
Work W	ML^2/T^2	N · m	ft-lb
		J (joule)	
Energy E	ML^2/T^2	N · m	ft-lb
		J (joule)	
Heat rate \dot{Q}	ML^2/T^3	J/s	Btu/s
Torque T	ML^2/T^2	N · m	ft-lb
Power P	ML^2/T^3	J/s	ft-lb/s
\dot{W}		W (watt)	
Viscosity μ	MLT	$N \cdot s/m^2$	lb-s/ft ²
Kinematic viscosity ν	L^2/T	m^2/s	ft ² /s
Mass flux \dot{m}	M/T	kg/s	slug/s
Flow rate Q	L^3/T	m^3/s	ft ³ /s
Specific heat c	$L^2/T^2\Theta$	J/kg · K	Btu/slug-°R
Conductivity K	$ML/T^3\Theta$	W/m · K	lb/s-°R

Dimensions, Units, and Physical Quantities

SI Prefixes

<i>Multiplication factor</i>	<i>Prefix</i>	<i>Symbol</i>
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi ^a	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

^aPermissible if used alone as cm, cm², or cm³.

Dimensions, Units, and Physical Quantities

A mass of 100 kg is acted on by a 400-N force acting vertically upward and a 600-N force acting upward at a 45° angle. Calculate the vertical component of the acceleration. The local acceleration of gravity is 9.81 m/s². (The rollers are frictionless.)

Solution

The first step in solving a problem involving forces is to draw a free-body diagram with all forces acting on it, as shown in Figure E1.1.

Next, apply Newton's second law (Eq. 1.2.4). It relates the net force acting on a mass to the acceleration and is expressed as

$$\Sigma F_y = ma_y$$

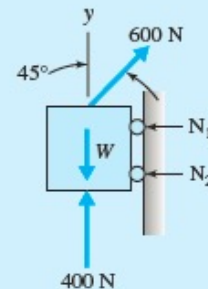


Figure E1.1

Using the appropriate components in the positive y -direction, with $W = mg$, we have

$$400 + 600 \sin 45^\circ - 100 \times 9.81 = 100a_y$$

$$a_y = \underline{-1.57 \text{ m/s}^2}$$

The negative sign indicates that the acceleration is in the negative y -direction, i.e., down.

Note: We have used only three significant digits in the answer since the information given in the problem is assumed known to three significant digits.

Continuum View of Gases and Liquids

- Substances can be both liquids or gases.
 - **Liquids** - Matter in which molecules are relatively free to change their positions with respect to each other. The molecules are restricted by cohesive forces so as to maintain a relatively fixed volume.
 - **Gas** – Matter in which molecules are unrestricted by cohesive forces. Gas has neither definite shape nor volume.

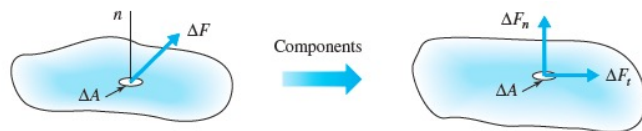


Figure 1.1 Normal and tangential components of a force.

- A force ΔF that acts on an area ΔA can be broken into tangential (F_t) and normal (F_n) components.
- **Stress** - Force divided by the area upon which it acts.

Continuum View of Gases and Liquids

- **Stress Vector** - The force vector divided by the area.
- **Normal Stress** - Normal component of force divided by the area.
- **Shear Stress (τ)** - Tangential force divided by the area. (defined as shown below)

$$\tau = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_t}{\Delta A}$$

Continuum View of Gases and Liquids

- **Microscopic Behavior of Fluids**
- Molecules are not stationary, but move about with high velocities.
- The molecules collide with each other and strike the walls of a container in which they are confined.
 - Gives rise to the pressure exerted by the gas.
- If volume increases (at a constant temperature):
 - Number of collisions (per unit area) decreases
 - Hence pressure decreases.
- If temperature increases:
 - Velocity of molecules increases
 - Hence pressure increases.

Continuum View of Gases and Liquids

- Assume that fluids act as a continuum:
 - A continuous distribution of a liquid or gas throughout a region of interest.

$$\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V}$$

- Density is used to find out if the continuum assumption is appropriate.
 - Δm : Incremental mass ΔV : Incremental volume
- Standard Atmospheric Conditions
 - Pressure: 101.3 kPa
 - Temperature: 15°C
 - Density of air: 1.23 kg/m³
 - Density of water: 1000 kg/m³

Continuum View of Gases and Liquids

- The continuum model can be checked for accuracy by comparing the characteristic length, l , with the mean free path.
- **Mean Free Path, λ : Average distance a molecule travels before it collides with another molecule.**
 - **If $l \gg \lambda$: The continuum model is acceptable.**

$$\lambda = 0.225 \frac{m}{\rho d^2}$$

Pressure and Temperature Scales

- The pressure, p , can be defined as:

$$p = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_n}{\Delta A}$$

ΔF_n : Incremental normal compressive force

ΔA : Incremental area

Units: N/m²

- **Absolute Pressure**: Zero is reached for an ideal vacuum.
- **Gage Pressure**: Pressure relative to the local atmospheric pressure.

$$P_{\text{absolute}} = P_{\text{atmospheric}} + P_{\text{gage}}$$

Pressure and Temperature Scales

- Temperature scales (Celsius and Fahrenheit scales)

$$K = ^\circ C + 273.15 \quad \text{Celsius to Kelvin}$$

$$^\circ R = ^\circ F + 459.67 \quad \text{Fahrenheit to Rankine}$$

	$^\circ C$	K
Steam point	100°	373
Ice point	0°	273
Special point	-18°	255
Absolute zero temperature	-273°	0°

Temperatures of special points.

Pressure and Temperature Scales

Example 1.2

A pressure gage attached to a rigid tank measures a vacuum of 42 kPa inside the tank shown in Figure E1.2, which is situated at a site in Colorado where the elevation is 2000 m. Determine the absolute pressure inside the tank.

Solution

To determine the absolute pressure, the atmospheric pressure must be known. If the elevation were not given, we would assume a standard atmospheric pressure of 100 kPa. However, with the elevation given, the atmospheric pressure is found from Table B.3 in Appendix B to be 79.5 kPa. Thus

$$p = -42 + 79.5 = \underline{37.5 \text{ kPa abs}}$$

Note: A vacuum is always a negative gage pressure.



Figure E1.2

Fluid Properties

Density and Specific Weight

- **Specific Weight, γ** : Weight per unit volume

- Units: N/m³

$$\gamma = \frac{W}{V} = \frac{mg}{V} = \rho g$$

g: Local gravity

- **Specific Gravity, S** : Ratio of density of a substance to the density of water at 4°C.

- Units: Dimensionless

$$S = \frac{\rho}{\rho_{\text{water}}} = \frac{\gamma}{\gamma_{\text{water}}}$$

Fluid Properties

Density and Specific Weight

Density, Specific Weight, and Specific Gravity of Air and Water at Standard Conditions

	<i>Density ρ</i>	<i>Specific weight γ</i>	<i>Specific gravity S</i>
	kg/m ³	N/m ³	
Air	1.23	12.1	0.00123
Water	1000	9810	1

Fluid Properties

Viscosity

- **Viscosity, μ** : Measure of the resistance of a fluid to gradual deformations by shear stress.
 - Accounts for energy losses in the transport of fluids in ducts or pipes
 - Plays a role in the generation of turbulence

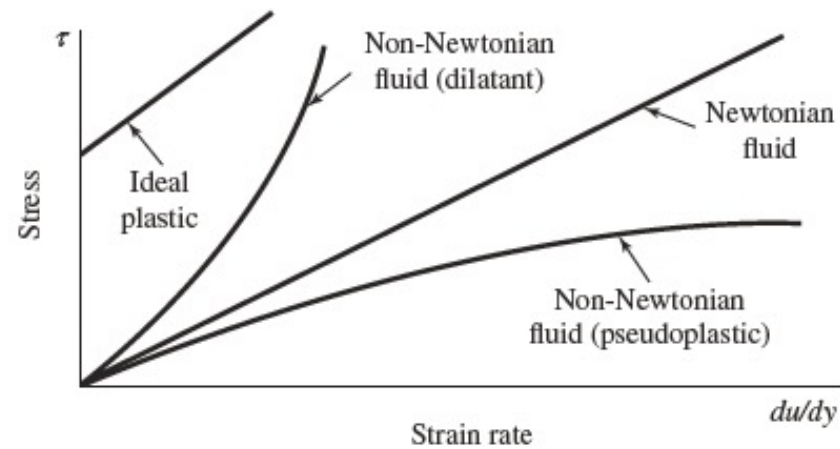
$$\tau = \mu \frac{du}{dy}$$

- Units: $\text{N}\cdot\text{s}/\text{m}^2$
- The differential, $\frac{du}{dy}$, is a velocity gradient (**strain rate**).
 - This is the rate at which a fluid element deforms.

Fluid Properties

Viscosity - Newtonian fluid

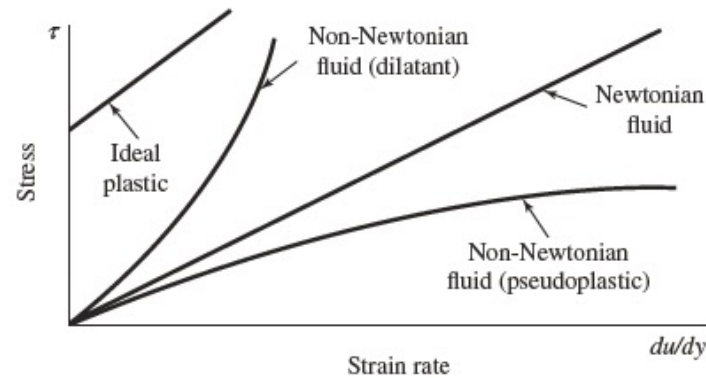
- **Newtonian fluid:** A fluid in which the shear stress is directly proportional to the velocity gradient.
 - E.g., Air, water, and oil



Newtonian and non-Newtonian fluids.

Fluid Properties

Viscosity - Non-Newtonian fluid

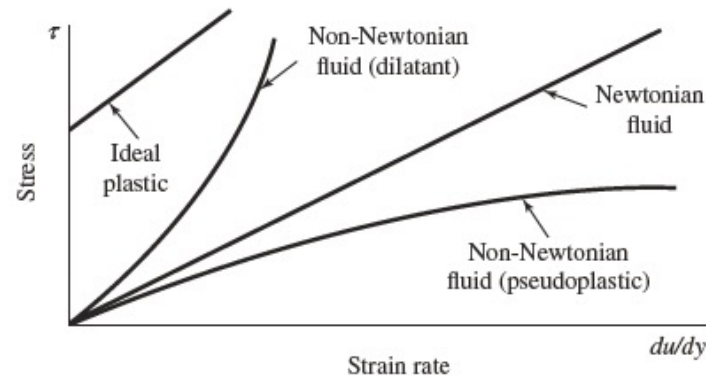


Newtonian and non-Newtonian fluids.

- **Dilatants** - Non-Newtonian fluids which become more resistant to motion as the strain rate increases.
 - E.g., Quicksand, slurries
- **Pseudoplastic** - A fluid which becomes less resistant to motion with increased strain rate.
 - E.g., Paint
- **Bingham fluids** - Require a minimum shear stress to cause motion.
 - E.g., Clay suspensions, toothpaste

Fluid Properties

Viscosity - Non-Newtonian fluid



Newtonian and non-Newtonian fluids.

- **No-slip Condition:** Causes fluid to adhere to the surface (due to viscosity)
- In equations, viscosity is often divided by density (kinematic viscosity).
- Units: m^2/s

$$v = \frac{\mu}{\rho}$$

Fluid Properties

Compressibility

- Can be described using the **Bulk modulus of elasticity, B**.
 - This is the ratio of the change in pressure to relative change in density.
 - Same units as pressure.
- For gases:
 - Significant changes in density (~4%) - Compressible.
 - Small density changes (under 3%) - Incompressible.

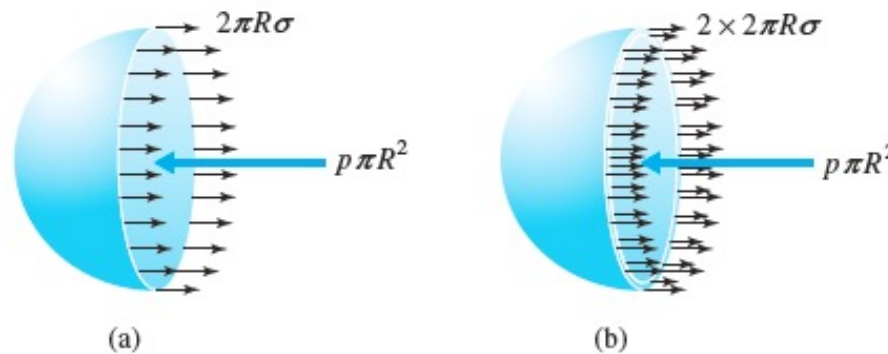
$$c = \sqrt{\left. \frac{\partial p}{\partial \rho} \right|_T} \cong \sqrt{\left. \frac{\Delta p}{\Delta \rho} \right|_T} = \sqrt{\frac{B}{\rho}}$$

- The speed of sound in a liquid can be found using the Bulk modulus of elasticity and density, as shown above.

Fluid Properties

Surface Tension

- Results from the attractive forces between molecules.
- Hence seen only in liquids at an interface (liquid-gas).
- Forces between molecules in a liquid bulk are equal in all directions.
 - No net force is exerted on them.
- At an interface, the molecules exert a force that has a resultant force.
 - Holds a drop of water on a rod and limits its size.

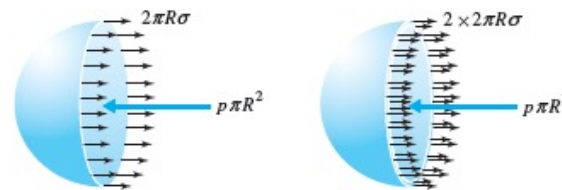


Internal forces in (a) a droplet and (b) a bubble.

Fluid Properties

Surface Tension

- Unit: Force per unit length, N/m
- Force results from the length (of fluid in contact with a solid) multiplied by the surface tension.
 - A droplet has one surface.
 - A bubble is a thin film of liquid with an inside and an outside surface.



$$p\pi R^2 = 2\pi R\sigma$$

$$\therefore p = \frac{2\sigma}{R}$$

Pressure in the droplet balances the surface tension around the circumference.

$$2 \times (2\pi R\sigma)$$

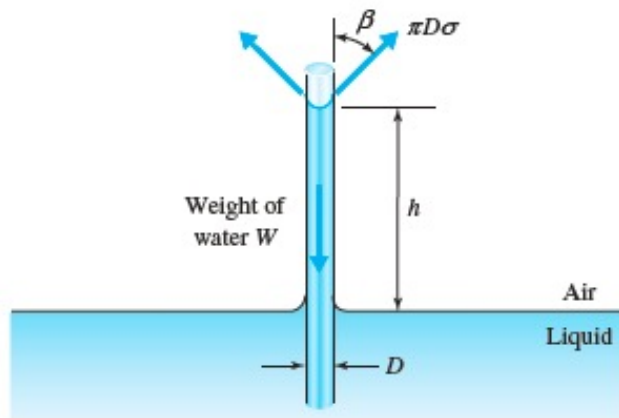
$$\therefore p = \frac{4\sigma}{R}$$

Pressure in the bubble is balanced by the surface tension forces on the two circumferences.

Fluid Properties

Surface Tension

- As seen before, the internal pressure in a bubble is twice as large as that in a droplet of a similar size.

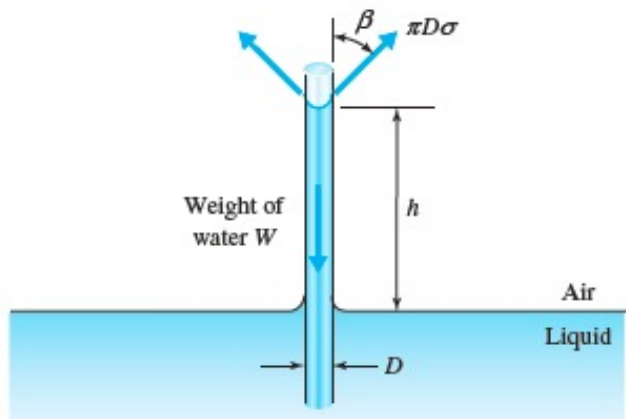


Rise in a capillary tube.

- Liquid rises in a glass capillary tube due to surface tension.
- The liquid makes a contact angle β with a glass tube.
 - For most liquids (and water) this is zero.
 - Mercury has an angle greater than 90° .

Fluid Properties

Surface Tension



Rise in a capillary tube.

$$\sigma \pi D \cos \beta = \gamma \frac{\pi D^2}{4} h$$

$$h = \frac{4\sigma \cos \beta}{\gamma D}$$

- h: Capillary rise
- D: Diameter
- p: Density
- σ : Surface tension

- 'h' can be determined by equating the vertical component of the surface tension force to the weight of the liquid column.

Fluid Properties

Surface Tension

A 2-mm-diameter clean glass tube is inserted in water at 15°C (Figure E1.4). Determine the height that the water will climb up the tube. The water makes a contact angle of 0° with the clean glass.

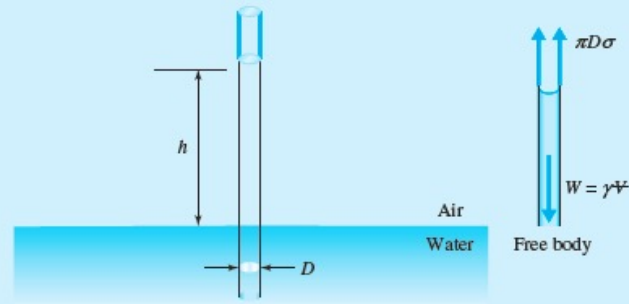


Figure E1.4

Solution

A free-body diagram of the water shows that the upward surface-tension force is equal and opposite to the weight. Writing the surface-tension force as surface tension times distance, we have

$$\sigma\pi D = \gamma \frac{\pi D^2}{4} h$$

or

$$h = \frac{4\sigma}{\gamma D} = \frac{4 \times 0.0741 \text{ N/m}}{9810 \text{ N/m}^3 \times 0.002 \text{ m}} = 0.01512 \text{ m} \quad \text{or} \quad \underline{15.12 \text{ mm}}$$

The numerical values for σ and ρ were obtained from Table B.1 in Appendix B. *Note:* If temperature is not given, the nominal value used for the specific weight of water is $\gamma = \rho g = 9810 \text{ N/m}^3$.

Fluid Properties

Vapor Pressure

- A certain fraction of a liquid will vaporize when a small quantity is placed in a closed container.
 - Will end when equilibrium between the liquid and gaseous states is reached.
- **Vapor Pressure:** Pressure resulting from molecules in a gaseous state.
 - E.g., Water at 15°C has a vapor pressure of 1.70 kPa absolute.
 - Depends on temperature (increases when temperature increases).
- **Boiling** occurs where vapor pressure equals atmospheric pressure.
- **Cavitation** is when bubbles form in a liquid when the local pressure falls below the vapor pressure of the liquid.
 - This is very damaging as these bubbles collapse in high-pressure regions.
 - Leads to pressure spikes (can damage ship's propellers, etc.).

Fluid Properties

Vapor Pressure

Calculate the vacuum necessary to cause cavitation in a water flow at a temperature of 80°C in Colorado where the elevation is 2500 m.

Solution

The vapor pressure of water at 80°C is given in Table B.1. It is 47.3 kPa absolute. The atmospheric pressure is found by interpolation using Table B.3 to be $79.48 - (79.48 - 61.64)500/2000 \cong 75.0$. The required pressure is then

$$p = 47.3 - 75.0 = -27.7 \text{ kPa} \quad \text{or} \quad \underline{27.7 \text{ kPa vacuum}}$$

Conservation Laws

- **SYSTEM:** Fixed quantity of matter upon which attention is focused.
- **NEWTON'S SECOND LAW:** The sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system.

Conservation Laws

- **CONSERVATION OF MASS:** Matter is indestructible.
- **CONSERVATION OF MOMENTUM:** [From Newton's Second Law] The momentum of a system remains constant if no external forces act on it.
- **CONSERVATION OF ENERGY:** The total energy of an isolated system remains constant.

Thermodynamic Properties and Relationships

- **Extensive Property** - Property that depends on the system's mass.
 - E.g., Momentum, Energy
- **Intensive Property** - Property that is independent of the system's mass.
 - E.g., Temperature, Pressure

Properties of an Ideal Gas

$$p = \rho RT$$

- Behavior of gases for most engineering applications can be described by the ideal gas law.
- For air, with temperatures more than -50°C and pressures not extremely high, the ideal gas law approximates the behavior of air to a good degree.

Thermodynamic Properties and Relationships

Properties of an Ideal Gas

$$p = \rho RT$$

- p : Absolute pressure
- ρ : Density
- T : Absolute temperature
- R : Gas constant

$$R = \frac{R_u}{M}$$

The gas constant is found using the universal gas constant and the molar mass.

$$R_u = 8.314 \text{ kJ/kmol} \cdot \text{K}$$

Thermodynamic Properties and Relationships

Properties of an Ideal Gas

$$pV = mRT$$

A tank with a volume of 0.2 m^3 contains 0.5 kg of nitrogen. The temperature is 20°C . What is the pressure?

Solution

Assume this is an ideal gas. Apply Eq. 1.7.1 (R can be found in Table B.4). Solving the equation, $p = \rho RT$, we obtain, using $\rho = m/V$,

$$p = \frac{0.5 \text{ kg}}{0.2 \text{ m}^3} \times 0.2968 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (20 + 273) \text{K} = \underline{218 \text{ kPa abs}}$$

Note: The resulting units are $\text{kJ/m}^3 = \text{kN} \cdot \text{m/m}^3 = \text{kN/m}^2 = \text{kPa}$. The ideal gas law requires that pressure and temperature be in absolute units.

Thermodynamic Properties and Relationships

First Law of Thermodynamics

- States that when a system changes from State 1 to State 2, its energy changes from E_1 to E_2 .
 - This energy change is **heat transfer** or **work**.
 - Heat Transfer **to** the system and work done **by** the system are **positive**.

$$Q_{1-2} - W_{1-2} = E_2 - E_1$$

- Q_{1-2} : Amount of heat transfer to the system.
- W_{1-2} : Amount of work done by the system.

Thermodynamic Properties and Relationships

First Law of Thermodynamics

$$E = m \left(\frac{V^2}{2} + gz + \tilde{u} \right)$$

- Energy (E) for the total energy consists of kinetic energy ($\frac{1}{2}mV^2$), potential energy (mgz), and internal energy ($m\tilde{u}$).
 - \tilde{u} : Internal energy per unit mass
- Work results from a force moving through a distance.
- If the force is because of pressure:

$$\begin{aligned} W_{1-2} &= \int_{l_1}^{l_2} F dl \\ &= \int_{l_1}^{l_2} pA dl = \int_{V_1}^{V_2} p dV \end{aligned}$$

Thermodynamic Properties and Relationships

A cart with a mass of 29.2 kg is pushed up a ramp with an initial force of 445 N (Figure E1.7). The force decreases according to

$$F = 72.95(6.1 - l) \text{ N}$$

If the cart starts from rest at $l = 0$, determine its velocity after it has traveled 6.1 m up the ramp. Neglect friction.

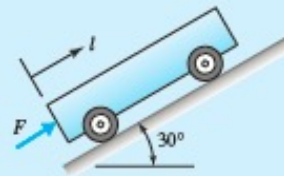


Figure E1.7

Solution

The energy equation (Eq. 1.7.6) allows us to relate the quantities of interest. Since there is no heat transfer, we have

$$-W_{1-2} = E_2 - E_1$$

Recognizing that the force is doing work on the system, the work is negative. Hence the energy equation becomes, using $W = \int F dl$,

$$-\left[-\int_0^{6.1} 72.95(6.1 - l) dl \right] = m \left(\frac{V_2^2}{2} + gz_2 \right) - m \left(\frac{V_1^2}{2} + gz_1 \right)$$

Taking the datum as $z_1 = 0$, we have $z_2 = 6.1 \sin 30^\circ = 3.05$ m. Thus

$$72.95 \times 6.1^2 - 72.95 \times \frac{6.1^2}{2} = 29.2 \left(\frac{V_2^2}{2} + 9.81 \times 3.05 \right)$$
$$\therefore V_2 = \underline{5.76 \text{ m/s}}$$

Note: We have assumed no internal energy change and no heat transfer.

Thermodynamic Properties and Relationships

Thermodynamic Quantities

- Enthalpy (H), created to help with thermodynamic calculations.

$$H = m\tilde{u} + pV$$

- Constant-pressure specific heat C_p and constant-volume specific heat C_v are used to calculate enthalpy and internal energy changes.

$$\Delta h = \int c_p dT \quad dh = c_p dT$$

$$\Delta \tilde{u} = \int c_v dT \quad d\tilde{u} = c_v dT$$

Thermodynamic Properties and Relationships

Thermodynamic Quantities

- **Ratio of specific heats (k):** The ratio of specific heats.

$$k = \frac{c_p}{c_v}$$

- A process in which pressure, temperature, and other properties are constant at any instant throughout the system is called a quasi-equilibrium process.
 - E.g., Compression/expansion in the cylinder of an internal combustion engine.
- **If no heat is transferred: Process is an isentropic process.**

Thermodynamic Properties and Relationships

Thermodynamic Quantities

- For an isentropic process:

$$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^k \quad \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{(k-1)/k} \quad \frac{T_1}{T_2} = \left(\frac{\rho_1}{\rho_2}\right)^{k-1}$$

- For a small pressure wave in a gas (at low frequency), the wave speed in an isentropic process is:

$$c = \sqrt{\left.\frac{dp}{d\rho}\right|_s} = \sqrt{kRT}$$

Thermodynamic Properties and Relationships

Thermodynamic Quantities

- For a small pressure wave in a gas (at a relatively high frequency):
 - Entropy is not constant.

$$c = \sqrt{\left. \frac{dp}{d\rho} \right|_T} = \sqrt{RT}$$

Thermodynamic Properties and Relationships

A cylinder fitted with a piston has an initial volume of 0.5 m^3 . It contains 2.0 kg of air at 400 kPa absolute. Heat is transferred to the air while the pressure remains constant until the temperature is 300°C . Calculate the heat transfer and the work done. Assume constant specific heats.

Solution

Using the first law (Eq. 1.7.6), and the definition of enthalpy, we see that, with no kinetic or internal energy changes, there results

$$\begin{aligned} Q_{1-2} &= p_2 V_2 - p_1 V_1 + m\tilde{u}_2 - m\tilde{u}_1 \\ &= m\tilde{u}_2 + p_2 V_2 - (m\tilde{u}_1 + p_1 V_1) \\ &= H_2 - H_1 = m(h_2 - h_1) = mc_p(T_2 - T_1) \end{aligned}$$

where Eq. 1.7.12 is used assuming c_p to be constant. The initial temperature is

$$T_1 = \frac{p_1 V_1}{mR} = \frac{400 \text{ kN/m}^2 \times 0.5 \text{ m}^3}{2.0 \text{ kg} \times 0.287 \text{ kJ/kg}\cdot\text{K}} = 348.4 \text{ K}$$

(Use $\text{kJ} = \text{kN}\cdot\text{m}$ to check the units.) Thus the heat transfer is (c_p is found in Table B.4)

$$Q_{1-2} = 2.0 \times 1.0[(300 + 273) - 348.4] = \underline{449 \text{ kJ}}$$

The final volume is found using the ideal gas law:

$$V_2 = \frac{mRT_2}{p_2} = \frac{2 \text{ kg} \times (0.287 \text{ kJ/kg}\cdot\text{K}) \times 573 \text{ K}}{400 \text{ kN/m}^2} = 0.822 \text{ m}^3$$

The work done for the constant-pressure process is, using Eq. 1.7.9 with $p = \text{const}$,

$$\begin{aligned} W_{1-2} &= p(V_2 - V_1) \\ &= 400 \text{ kN/m}^2(0.822 - 0.5) \text{ m}^3 = 129 \text{ kN}\cdot\text{m} \text{ or } \underline{129 \text{ kJ}} \end{aligned}$$

Thermodynamic Properties and Relationships

The temperature on a cold winter day in the mountains of Wyoming is -30°C at an elevation of 4 km. Calculate the density of the air assuming the same pressure as in the local atmosphere; also find the speed of sound.

Solution

From Table B.3 we find the atmospheric pressure at an elevation of 4 km to be 61.64 kPa. The absolute temperature is found to be

$$T = -30 + 273 = 243 \text{ K}$$

Using the ideal gas law, the mass density is calculated as

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{61.64}{0.287 \times 243} = \underline{0.884 \text{ kg/m}^3}\end{aligned}$$

The speed of sound, using Eq. 1.7.17, is determined to be

$$\begin{aligned}c &= \sqrt{kRT} \\ &= \sqrt{1.4 \times 287 \times 243} = \underline{312 \text{ m/s}}\end{aligned}$$

Note: The gas constant in the foregoing equations has units of $\text{J/kg} \cdot \text{K}$ so that the appropriate units result.

Summary

- To relate units, Newton's second law is used:
 - $N = \text{kg}\cdot\text{m}/\text{s}^2$
- When making calculations, the answer should have the same number of significant digits as the **least** accurate number used in the calculations.
- Pressure is expressed as gage pressure unless stated otherwise.
- The density, or specific weight, of a fluid can be found if the specific gravity is given:

$$\rho_x = S_x \rho_{\text{water}} \quad \gamma_x = S_x \gamma_{\text{water}}$$

- The shear stress due to viscous effects in a simple flow is:

$$\tau = \mu \frac{du}{dy}$$